LXXXVI. *On Relativistic Cosmology.* By H. P. Robertson, *Ph.D., National Research Fellow in Mathematics**.

THE general theory of relativity considers physical space-time as a four-dimensional manifold whose line element coefficients $g_{\mu\nu}$ satisfy the differential equations

$$G_{\mu\nu} = \lambda g_{\mu\nu} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in all regions free from matter and electromagnetic field, where $G_{\mu\nu}$ is the contracted Riemann-Christoffel tensor associated with the fundamental tensor $g_{\mu\nu}$, and λ is the cosmological constant †. An "empty world," *i. e.* a homogeneous manifold at all points of which equations (1) are satisfied, has, according to the theory, a constant Riemann curvature, and any deviation from this fundamental solution is to be directly attributed to the influence of matter or energy. In considerations involving the nature of the world as a whole the irregularities caused by the aggregation of matter into stars and stellar systems may be ignored; and if we further assume that the total matter in the world has but little effect on its macroscopic properties, we may consider them as being determined by the solution for an empty world ‡.

The solution of (1), which represents a homogeneous manifold, may be written in the form:

$$ds^2 = -\frac{d\rho^2}{1-\kappa^2\rho^2} - \rho^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-\kappa^2\rho^2)c^2d\tau^2, \quad (2)$$

where $\kappa = \sqrt{\lambda/3}$. If we consider ρ as determining distance from the origin (the measured distance being then $\sin^{-1}\kappa\rho/\kappa$) and τ as measuring the proper-time of a clock at the origin, we are led to the de Sitter spherical world; the astronomical

* Communicated by Prof. A. S. Eddington, F.R.S.

† A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Ann. d. Physik*, xlix. p. 769 (1916); 'Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie,' Berlin, Sitzungsberichte, 1917, p. 142; W. de Sitter, "On Einstein's Theory of Gravitation and its Astronomical Consequences." Monthly Notices, R. A. S. lxxvi. p. 699, lxxvii. p. 155, lxxviii. p. 3 (1916–17). The notation here used is that of A. S. Eddington, 'The Mathematical Theory of Relativity,' Cambridge, 1923.

‡ This view, due to de Sitter, is an alternative to one proposed by Einstein in which the existing matter is considered uniformly spread throughout the world. The equations (1) are not applicable to this latter case, as terms involving the density must then be added to the right-hand side.

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